Valuation Models

Bonds, Preferred Stock
and Common Stock

Market Value

Market value of any asset, whether it be a bond, a share of preferred or common stock, a rare painting or a classic car, is theoretically the discounted value of the expected cash flows

Valuation Models - Bonds

AT&T has a bond issue outstanding:

- Coupon rate = 8%/yr comp semiannually
- Matures in 20 years
- Par value = face value = principal = 1,000

Calculate its market value

One additional piece of information is needed

Synonyms

- **interest rate**
- **yield to maturity** on comparable securities
- market rate of return or market yield
- going rate of return

Given i, find P_0

- Let's assume that the yield to maturity or interest rate on similar bonds is 10%/yr compounded semiannually
- P_0 is the discounted value of the expected cash flows
- P_0 is the discounted value of the annuity of coupon payments and the return of principal at maturity

Value of the AT&T bond

$$C = \frac{.08 \times 1000}{2} = 40\text{/period}, \quad n = 20 \times 2 = 40 \text{ periods} \quad \text{and } i = \frac{.10}{2} = .05\text{/period}$$

$$P_0 = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{1000}{(1+i)^n}$$

$$P_0 = \frac{40}{(1.05)^1} + \frac{40}{(1.05)^2} + \cdots + \frac{40}{(1.05)^{40}} + \frac{1000}{(1.05)^{40}}$$

$$P_0 = 40(PVIFa - 5\% - 40) + \frac{1000}{(1.05)^{40}}$$

$40 \Rightarrow$ PMT $\quad 5 \Rightarrow$ I/yr $\quad 40 \Rightarrow$ n $\quad 1000 \Rightarrow$ FV

PV = 828.41 < 1000 Bond sells at a discount

Yield to Maturity on the AT&T bond

$C=\frac{.08\times1000}{2}=40\text{/period},\ \ n=20\times2=40\text{ periods and }P=\$828.41$

$$P_0=\frac{C}{(1+i)^1}+\frac{C}{(1+i)^2}+\cdots+\frac{C}{(1+i)^n}+\frac{1000}{(1+i)^n}$$

$$828.41=\frac{40}{(1+i)^1}+\frac{40}{(1+i)^2}+\cdots+\frac{40}{(1+i)^{40}}+\frac{1000}{(1+i)^{40}}$$

$$828.41=40(PVIFa-i\%-40)+\frac{1000}{(1+i)^{40}}$$

$40\Rightarrow$ PMT -828.41 $\Rightarrow$ PV 40 $\Rightarrow$ n 1000 $\Rightarrow$ FV

$i=.05/period$ or $i=.10/yr$ compounded semiannually

Internal Rate of Return (IRR)

$$CF_0=\frac{CF_1}{(1+r)^1}+\frac{CF_2}{(1+r)^2}+\cdots+\frac{CF_n}{(1+r)^n}=\sum_{t=1}^{n}\frac{CF_t}{(1+r)^t}$$

CF_t = cash flow, end of period t

n = life of the project

r = IRR

Why the discount?

- New 20 year securities being issued today probably are paying a coupon of $100/yr
- If it did sell for $1,000 it would yield only 8% which is less than other similar bonds
- Price must adjust to bring the yield or interest rate into line with similar bonds

Back in time

- It's common for a new bond to be issued at a price close to its par value of 1,000
- 5 years ago AT&T issued our bonds with a maturity of 25 years and an annual coupon of 8%. Let's assume that the interest rate at that time was 8.2%/yr, compounded semiannually. What was the issuing price?

Issuing price of AT&T bonds

$$C = \frac{.08 \times 1000}{2} = 40/\text{period}, \quad n = 25 \times 2 = 50 \text{ periods and } i = \frac{.082}{2} = .041/\text{period}$$

$$P_0 = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{1000}{(1+i)^n}$$

$$P_0 = \frac{40}{(1.041)^1} + \frac{40}{(1.041)^2} + \cdots + \frac{40}{(1.041)^{50}} + \frac{1000}{(1.041)^{50}}$$

$$P_0 = 40(PVIFa - 4.1\% - 50) + \frac{1000}{(1+i)^{50}}$$

$40 \Rightarrow$ PMT $4.1 \Rightarrow$ I/yr $50 \Rightarrow$ n $1000 \Rightarrow$ FV

PV = 978.88

Can you say "capital loss?"

- What about the investor who bought these very safe bonds 5 years ago and now wants to sell?
- Can she recover her $978.88?
- No, only $828.41 because interest rates have risen
- Let's see an old slide again

Incredibly important relationships

$$i \Uparrow \Leftrightarrow P_{bonds} \Downarrow$$

$$i \Downarrow \Leftrightarrow P_{bonds} \Uparrow$$

Why the inverse relationship?

$$P_0 \downarrow = \frac{\overline{C}}{(1+i\uparrow)^1} + \frac{\overline{C}}{(1+i\uparrow)^2} + \cdots + \frac{\overline{C}}{(1+i\uparrow)^n} + \frac{\overline{1000}}{(1+i\uparrow)^n}$$

With the numerators fixed (bonds are called "fixed income" securities), if the denominator changes, the only thing that can give is that the left side of the equation has to move in the opposite direction.

What if interest rate had fallen?

$$\text{C} = \frac{.08 \times 1000}{2} = 40\text{/period}, \quad \text{n} = 20 \times 2 = 40 \text{ periods} \quad \text{and i} = \frac{.05}{2} = .025\text{/period}$$

$$P_0 = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{1000}{(1+i)^n}$$

$$P_0 = \frac{40}{(1.025)^1} + \frac{40}{(1.025)^2} + \cdots + \frac{40}{(1.025)^{40}} + \frac{1000}{(1.025)^{40}}$$

$$P_0 = 40(PVIFa - 2.5\% - 40) + \frac{1000}{(1.025)^{40}}$$

40 ⇒ PMT 2.5 ⇒ I/yr 40 ⇒ n 1000 ⇒ FV

PV = 1376.54 > 1000 Bond sells at a premium

Why the premium?

- New 20 year securities being issued today probably are paying a coupon of $50/yr
- If it did sell for $1,000 it would yield 8% which is more than other similar bonds
- Price must adjust to bring the yield or interest rate into line with similar bonds

Rates of return over 5 year period

$$P_0 = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \cdots + \frac{C}{(1+r)^n} + \frac{FV}{(1+r)^n} \quad r = IRR$$

$$978.88 = \frac{40}{(1+r)^1} + \frac{40}{(1+r)^2} + \cdots + \frac{40}{(1+r)^{10}} + \frac{828.41}{(1+r)^{10}}$$

$$978.88 = 40(PVIFa\ -r\%-10) + \frac{828.41}{(1+r)^{10}}$$

40 ⇒ PMT 10 ⇒ n 828.41 ⇒ FV -978.88 ⇒ PV

r = I/yr = 2.73%/period = 5.46%/yr comp semiannually

$$978.88 = \frac{40}{(1+r)^1} + \frac{40}{(1+r)^2} + \cdots + \frac{40}{(1+r)^{10}} + \frac{1376.54}{(1+r)^{10}}$$

$$978.88 = 40(PVIFa\ -r\%-10) + \frac{1376.54}{(1+r)^{10}}$$

40 ⇒ PMT 10 ⇒ n 1376.54 ⇒ FV - 978.88 ⇒ PV

r = I/yr = 7.02%/period = 14.04%/yr comp semiannually

Important observations

- The longer the period to maturity, the more sensitive is a bond’s price to a given change in the interest rate
- Maturity is one factor affecting a bond’s yield
- Long-term bonds are inherently riskier than short-term bonds and generally carry higher yields to maturity
- Especially true when rates are low and expected to rise

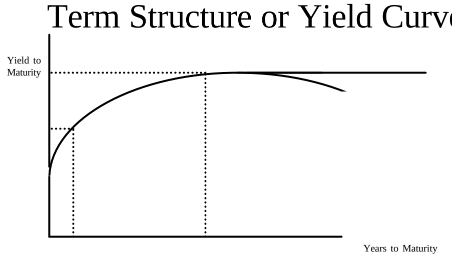
Maturity vs. yield example

- Bond A: i_A=5%, C_A=5%, n_A=2, P_A=1000
- Bond B: i_B=5%, C_B=5%, n_B=20, P_B=1000
- Instantaneously change $i_A = i_B$=8%
- Verify that P_A=945.55 and P_B=703.11
- 3 percentage point rise in interest rate produces a much bigger decrease in price of long-term bond B than in short-term bond A

Interest rate risk

- Interest rate risk is the possibility that the interest rate will rise and the price of bonds will fall.
- The price of long-term bonds will fall more than the price of short-term bonds for a given change in the interest rate
- Relationship between maturity and yield is shown by the term structure of interest rates

Term Structure or Yield Curve



Preferred Stock

- Perpetual – infinite maturity
- Constant fixed dividend – never changes
- Par value usually $50 or $100/share
- If dividend rate=8% and par=$50, D=.08x50 = $4.00/share
- k_p ➔ market capitalization or discount rate for a share of preferred stock of the given risk class

Preferred Stock Valuation

$$P_{pfd} = \frac{D_1}{(1+k_P)^1} + \frac{D_2}{(1+k_P)^2} + \cdots + \frac{D_\infty}{(1+k_P)^\infty} = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_P)^t}$$

For preferred stock, $D_1 = D_2 = \cdots = D_\infty$

$$P_{pfd} = \frac{D}{k_P}$$

Common Stock

- Why buy a share of common stock?
  - Capital gains
  - Dividend stream
- Let's consider an **arbitrary** 10 year holding period

Common Stock Valuation

$$P_0 = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \cdots + \frac{D_{10}}{(1+k_e)^{10}} + \frac{P_{10}}{(1+k_e)^{10}}$$

P_{10} = price per share paid by second investor at end of year 10

$$P_{10} = \frac{D_{11}}{(1+k_e)^1} + \frac{D_{12}}{(1+k_e)^2} + \cdots + \frac{D_n}{(1+k_e)^{n-10}} + \frac{P_n}{(1+k_e)^{n-10}}$$

could have a third investor, but assume $n \Rightarrow \infty$

$$P_0 = \frac{D_1}{(1+k_e)^1} + \cdots + \frac{D_{10}}{(1+k_e)^{10}} + \frac{1}{(1+k_e)^{10}}\left[\frac{D_{11}}{(1+k_e)^1} + \frac{D_{12}}{(1+k_e)^2} + \cdots + \frac{D_\infty}{(1+k_e)^\infty}\right]$$

$$P_0 = \frac{D_1}{(1+k_e)^1} + \cdots + \frac{D_{10}}{(1+k_e)^{10}} + \frac{D_{11}}{(1+k_e)^{11}} + \frac{D_{12}}{(1+k_e)^{12}} + \cdots + \frac{D_\infty}{(1+k_e)^\infty}$$

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^t}$$ This equation is basis for all common stock valu ation

Common Stock Growth Models

- Need to make assumptions regarding the behavior of the dividend stream
- Normal growth model – describing large majority of firms
- "Super" growth model – describing the exceptional firms

Normal Growth Model

Assume dividends will grow at an annual rate of g for an infinite duration

g is roughly equal to the nominal growth of the economy ≈ 4 to 8%

$D_1 = D_0(1+g)^1$

$D_2 = D_1(1+g)^1 = D_0(1+g)^2$

$\vdots \quad \vdots \quad \vdots$

$D_n = D_{n-1}(1+g)^1 = D_0(1+g)^n$

$$P_0 = \lim_{n\to\infty}\left[\frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \cdots + \frac{D_n}{(1+k_e)^n}\right]$$

$$P_0 = \lim_{n\to\infty}\left[\frac{D_0(1+g)^1}{(1+k_e)^1} + \frac{D_0(1+g)^2}{(1+k_e)^2} + \cdots + \frac{D_0(1+g)^n}{(1+k_e)^n}\right]$$

if $n \to \infty$ and $k_e > g$

$$P_0 = \frac{D_1}{(k_e - g)}$$

Normal Growth Model

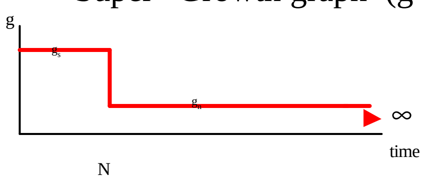
if $n \to \infty$ and $k_e > g$

$$P_0 = \frac{D_1}{(k_e - g)}$$

Example : $D_1 = \$2.00, k_e = 14\%, g = 4\%$

Then $P_0 = \frac{2.00}{(.14 - .04)} = \$20.00/\text{share}$

"Super" Growth graph (g>=k_e)



Assumption: earnings and dividends grow at super rate g_s for N years before declining to a normal growth rate of g_n for indefinite future

Analyze as if stock is held for N years then sold for P_N

"Super" Growth Model

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^t}$$

$$P_0 = \sum_{t=1}^{N} \frac{D_t}{(1+k_e)^t} + \frac{P_N}{(1+k_e)^N}$$

$$P_0 = \sum_{t=1}^{N} \frac{D_0(1+g_s)^t}{(1+k_e)^t} + \frac{P_N}{(1+k_e)^N}$$

P_N = PV of dividends N +1 to ∞ discounted to time N

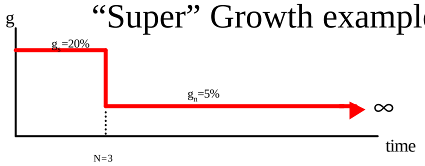
$$P_0 = \sum_{t=1}^{N} \frac{D_0(1+g_s)^t}{(1+k_e)^t} + \frac{1}{(1+k_e)^N}\left[\frac{D_{N+1}}{(1+k_e)^1} + \frac{D_{N+2}}{(1+k_e)^2} + \frac{D_{N+3}}{(1+k_e)^3} + \cdots + \infty\right]$$

$$P_0 = \sum_{t=1}^{N} \frac{D_0(1+g_s)^t}{(1+k_e)^t} + \frac{1}{(1+k_e)^N}\left[\frac{D_N(1+g_n)^1}{(1+k_e)^1} + \frac{D_N(1+g_n)^2}{(1+k_e)^2} + \frac{D_N(1+g_n)^3}{(1+k_e)^3} + \cdots + \infty\right]$$

$$P_0 = \sum_{t=1}^{N} \frac{D_0(1+g_s)^t}{(1+k_e)^t} + \frac{1}{(1+k_e)^N}\left[\sum_{t=1}^{\infty} \frac{D_N(1+g_n)^t}{(1+k_e)^t}\right] \quad k_e > g_n \text{ and } n \to \infty$$

$$P_0 = \sum_{t=1}^{N} \frac{D_0(1+g_s)^t}{(1+k_e)^t} + \frac{1}{(1+k_e)^N}\left[\frac{D_{N+1}}{k_e - g_n}\right]$$

"Super" Growth example



$$P_0 = \sum_{t=1}^{N} \frac{D_0(1+g_s)^t}{(1+k_e)^t} + \frac{P_N}{(1+k_e)^N} \quad \text{where } P_N = \frac{D_{N+1}}{k_e - g_n}$$

Example: $D_0 = \$1.00,\ g_s = 20\%,\ g_n = 5\%,\ k_e = 15\%,\ N = 3$

$$P_0 = \frac{1.00(1.20)^1}{(1.15)^1} + \frac{1.00(1.20)^2}{(1.15)^2} + \frac{1.00(1.20)^3}{(1.15)^3} + \frac{P_3}{(1.15)^3}$$

$$P_3 = \frac{D_4}{k_e - g_n} = \frac{(1.00)(1.20)^3(1.05)^1}{.15-.05} = 18.14$$

$$P_0 = 3.27 + \frac{18.14}{(1.15)^3} = 15.20$$

"Super" Growth exam question

Find P_1 and P_{30}

$$P_1 = \frac{D_2}{(1+k_e)^1} + \frac{D_3}{(1+k_e)^2} + \frac{P_3}{(1+k_e)^2}$$

$$P_1 = \frac{1.44}{(1.15)^1} + \frac{1.73}{(1.15)^2} + \frac{18.14}{(1.15)^2}$$

$$P_1 = \$16.28$$

$$P_{30} = \frac{D_{31}}{k_e - g_n} = \frac{1.00(1.20)^3(1.05)^{28}}{.15-.05} = \$67.74$$